Hardness Results for Vertex-Edge Roman Domination in Graphs

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Abstract

The vertex-edge Roman dominating function is defined as a function $f : V(G) \to \{0, 1, 2\}$ in which for each edge $e = uv$ of G either $max\{f(u), f(v)\} \neq 0$ or there exists a vertex w such that either $wu \in E$ or $wv \in E$ and $f(w) = 2$. The weight of the vertex-edge Roman dominating function, denoted as $g(V)$, is defined as $g(V) = \sum_{u \in V} g(u)$. The vertex-edge Roman domination number of G, denoted as $\gamma_{veR}(G)$, is the minimum weight of a vertex-edge Roman dominating function of G. The minimum vertex-edge Roman domination problem (Min-VERDP) is to find a vertex-edge Roman dominating function of minimum weight. The decision version of Min-VERDP is known to be NP-complete for bipartite graphs. We strengthen this result by proving that this problem remains NP-complete for perfect edge elimination bipartite graphs. Next, we show that the Min-VERDP cannot be approximated within $(1 - \epsilon)$ ln $|V|$ for any $\epsilon > 0$ unless $NP \subseteq DTIME(|V|^{O(ln(ln|V|))})$. Finally, we show that the Min-VERDP is APX-complete for graphs with maximum degree 5.

Keywords: Vertex-edge Roman domination, Roman domination, NP-Complete, Approximation result, APX - Complete

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